FALL 2019: MATH 558 QUIZ 2 SOLUTIONS

Use the First Principle of Mathematical Induction to prove:

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6},$$

for all $n \geq 1$. Be sure to identity the base case and the inductive hypothesis.

Solution. The base case, $n = 1$: $1 = \frac{1(1+1)(2\cdot 1+1)}{6} = \frac{6}{6} = 1$.

The inductive step: Assume

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6},$$

for $n \geq 1$ and use this to prove the $n+1$ case. We add $(n+1)^2$ to both sides of the equation above to get:

$$\begin{aligned}
1^2 + 2^2 + 3^2 + \cdots + n^2 + (n+1)^2 &= \frac{n(n+1)(2n+1)}{6} + (n+1)^2 \\
&= \frac{n(n+1)(2n+1)}{6} + \frac{(n+1)^2}{6} \\
&= \frac{n(n+1)(2n+1) + 6(n+1)^2}{6} \\
&= \frac{(n+1)\{n(2n+1) + 6(n+1)\}}{6} \\
&= \frac{(n+1)\{2n^2 + 7n + 6\}}{6} \\
&= \frac{(n+1)(n+2)(2n+3)}{6} \\
&= \frac{(n+1)((n+1)+1)(2(n+1)+1)}{6}.
\end{aligned}$$